Int. J. Solids Structures, 1972, Vol. 8, pp. 1373 to 1387. Pergamon Press. Printed in Great Britain

CLASSICAL AND NONLINEAR BUCKLING ANALYSES OF SPHERICAL SANDWICH SHELLS

NURI AKKAS†

Department of Civil Engineering, Clemson University, Clemson, South Carolina

Abstract—The buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with dissimilar face sheets under a uniform pressure is studied. The numerical results show that the buckling and initial post-buckling behavior of clamped shallow spherical sandwich shells with dissimilar face sheets is similar to that of the corresponding homogeneous shell.

The classical buckling analysis for spherical sandwich shells under a uniform pressure is also presented. The results indicate that it is possible to obtain the buckling curves of spherical sandwich caps from those of the homogeneous cap using a magnification factor which is obtained via the classical buckling analysis, for large values of the sandwich shell parameter.

INTRODUCTION

THE purpose of this investigation is twofold. Firstly, the buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with dissimilar face sheets is studied. The effects of the core thickness and core shear modulus on the imperfection sensitivity of the sandwich cap are investigated. The analysis of the behavior of the shell immediately after bifurcation buckling is based on Koiter's initial postbuckling theory [1]. It is essentially a perturbation technique which relies on the principle of stationary potential energy. This technique is transcribed by Fitch [2] into a form suitable for application to the shallow spherical homogeneous shell.

Asymmetric buckling behavior of clamped shallow spherical homogeneous shells under a uniform pressure was studied by Huang [3]. The initial postbuckling analysis for this problem has been provided by Fitch and Budiansky [4]. Akkas and Bauld [5] presented a sequence of boundary value problems that are relevant to the analysis of the buckling and initial postbuckling behavior of shallow spherical sandwich shells with dissimilar face sheets under certain axisymmetrical loads. The numerical results presented in Ref. [5] for the clamped shallow spherical sandwich shell with similar face sheets subjected to a uniform pressure show that the buckling and initial post-buckling behavior of the sandwich cap is similar to that of the classical homogeneous cap [3, 4]. The axisymmetric snap-through buckling behavior of the spherical sandwich cap with some different material parameters is studied by Akkas [6]. The results of Ref. [5, 6] are limited to the sandwich shells with face sheets made from the same materials and whose thicknesses are equal. For various reasons, such as the optimization of sandwich structures subject to thermal as well as mechanical loads, it is often necessary to have facings of different thicknesses and/or materials. It is

† Assistant Professor. Present Address: Department of Civil Engineering, Middle East Technical University, Ankara, Turkey.

the purpose of this investigation to study the buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with face sheets of different thicknesses and/or materials subjected to a uniform pressure.

The second purpose of the investigation is to determine the relationship between the buckling behaviors of spherical sandwich and homogeneous shells, which were shown to be strikingly similar in Ref. [5, 6], at least for the specific sandwich shell parameter considered. To achieve this purpose, the classical (linear) buckling behavior of a complete sandwich sphere will be studied. In this respect, the present work is an application of Hutchinson's work [7] to the sandwich sphere.

SANDWICH SHELL WITH DISSIMILAR FACE SHEETS

Theory

The development of the set of nonlinear differential equations that is suitable for the analysis of the buckling and initial postbuckling behavior of thin shallow spherical sandwich shells under axisymmetrical loads has been presented by Akkas and Bauld [5]. The behavior of thin clamped shallow spherical sandwich shells under pressures that are distributed uniformly over the entire reference surface of the shell undergoing moderately large deflections can be described by the nondimensional equations:

$$
\begin{aligned}
2\nabla^4 w = {} & \nabla^2 f + \left(\frac{1}{x}f' + \frac{1}{x^2}\ddot{f}\right)w'' + \left(\frac{1}{x}w' + \frac{1}{x^2}\ddot{w}\right)f'' \\
& - 2\left(\frac{1}{x}\dot{f}' - \frac{1}{x^2}\dot{f}\right)\left(\frac{1}{x}\dot{w}' - \frac{1}{x^2}\dot{w}\right) \\
& - \Gamma_1\left(\alpha' + \frac{1}{x}\alpha + \frac{1}{x}\dot{\beta} - \nabla^2 w\right) + 4p,
\end{aligned} \tag{1}
$$

$$
\frac{1}{2}\nabla^4 f = -\nabla^2 w + \left(\frac{1}{x}\dot{w}' - \frac{1}{x^2}\dot{w}\right)^2 - \left(\frac{1}{x}w' + \frac{1}{x^2}\ddot{w}\right)w'', \tag{2}
$$

$$
\frac{(1-\nu)}{2}\nabla^2\alpha + \frac{(1+\nu)}{2}\left(\alpha'' + \frac{1}{x}\alpha' + \frac{1}{x}\dot{\beta}'\right) - \frac{1}{x^2}\alpha - \frac{(3-\nu)}{2x^2}\dot{\beta} - \Gamma_2(\alpha - w') = 0, \tag{3}
$$

$$
\frac{(1-\nu)}{2}\nabla^2\beta + \frac{(1+\nu)}{2}\left(\frac{1}{x}\dot{\alpha}' + \frac{1}{x^2}\ddot{\beta}\right) + \frac{(3-\nu)}{2x^2}\dot{\alpha} - \frac{(1-\nu)}{2x^2}\beta - \Gamma_2\left(\beta - \frac{1}{x}\dot{w}\right) = 0, \tag{4}
$$

with boundary conditions at the clamped edge ($x = \lambda$):

$$
w = 0, \tag{5}
$$

$$
w' = 0, \tag{6}
$$

$$
f'' - \frac{\nu}{\lambda}f' - \frac{\nu}{\lambda^2}\ddot{f} = 0, \tag{7}
$$

$$\lambda\left(f''-\frac{v}{x}f'-\frac{v}{x^2}\ddot{f}\right)'-\frac{1}{\lambda}f'-\frac{1}{\lambda^2}\ddot{f}+v\ddot{f}''+2(1+v)\left(\frac{1}{x}\ddot{f}\right)'=0, \tag{8}$$

$$\alpha = 0, \tag{9}$$

$$\beta = 0. \tag{10}$$

The nondimensional quantities appearing in equations (1)–(10) are related to the corresponding physical quantities through the relations

$$\left.\begin{aligned}
&x = \frac{\lambda}{a}r, \qquad \lambda = 2[3(1-v^2)]^{1/4}\left(\frac{H}{t_1}\right)^{1/2}\left(\frac{1+\varepsilon\mu}{1+\varepsilon\mu^3}\right)^{1/4},\\
&w = \frac{\lambda^2}{2H}W, \qquad \alpha = \frac{\lambda a}{2H}\bar{\alpha}, \qquad \beta = \frac{\lambda a}{2H}\bar{\beta},\\
&f = \frac{\lambda^4 F}{2H^2E_1t_1(1+\varepsilon\mu)}, \qquad p = \frac{a^4\lambda^2 q}{16E_1t_1H^3}(1+\varepsilon\mu)^{-1},\\
&\Gamma_1 = \frac{\Lambda\lambda^2(1+\mu)^2}{8\delta(1+\varepsilon\mu)}[1+2\delta/(1+\mu)]^2,\\
&\Gamma_2 = \frac{\Lambda\lambda^2(1+\varepsilon\mu^3)}{48\varepsilon\mu\delta}, \qquad \Lambda = \left(\frac{G}{E_1}\right)\left(\frac{a}{H}\right)^2,\\
&\varepsilon = E_2/E_1, \qquad \mu = t_2/t_1, \qquad \delta = c/t_1.
\end{aligned}\right\} \tag{11}$$

The variables t_1, t_2, c, a and H are defined in Fig. 1. The transverse shearing modulus associated with the core material is denoted by G, while E_1 and E_2 signify the moduli of elasticity of the upper and lower face sheets, respectively. The Poisson's ratio for the face sheets is denoted by v (Poisson's ratios for the two face sheets are assumed to be equal) and q is the intensity of the uniform pressure. The radial and circumferential coordinates are denoted by r and ϕ, respectively. The quantities w, f, α, β are the nondimensional transverse deflection, Airy stress function and shear angles, respectively; $W, F, \bar{\alpha}, \bar{\beta}$ are the corresponding conventional physical quantities as described in Ref. [5]. The primes and dots represent, respectively, derivatives with respect to x and ϕ; and ∇^2 is the Laplacian operator in polar coordinates.

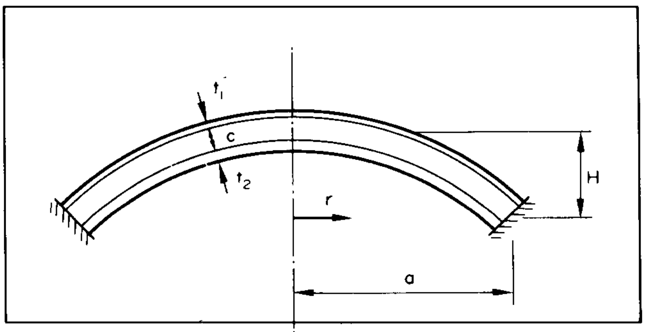

FIG. 1. Geometry of a clamped shallow spherical sandwich shell.

Since the derivation of the boundary value problems governing the axisymmetric buckling, bifurcation buckling and initial postbuckling behavior of the shell has already been presented in Ref. [5], it will not be repeated here. However, for a better understanding of the numerical results that will be presented later, it is necessary to describe the initial postbuckling coefficient b and the initial postbuckling relative stiffness parameter ϑ.

The theory of initial postbuckling behavior, originated by Koiter [1], leads, via an asymptotically valid calculation, to the determination of the initial slope of the bifurcation path. This result can then be used to predict whether buckling of the structure is sensitive to small initial geometric imperfections. The significance of the initial postbuckling coefficient b is connected with the notions of imperfection-sensitive structures. It has been shown in Ref. [2, 4] that structures containing initial geometric imperfections are imperfection-sensitive, in the sense that the buckling load for the imperfect structure should be expected to be less than that for the corresponding perfect structure, whenever the load for the perfect structure initially decreases at the bifurcation point, Fig. 2(a). A structure is said to be imperfection-insensitive, in the sense that load-deflection curve for the imperfect structure exhibits a much milder growth of displacement as the load reaches and exceeds the classical buckling load of the corresponding perfect structure, whenever the load for the perfect structure increases subsequent to asymmetrical buckling, Fig. 2(b). Accordingly, a structure is said to be imperfection-sensitive or imperfection-insensitive according to whether the initial postpuckling coefficient b is negative or positive, respectively.

The initial postbuckling relative stiffness parameter ϑ, which was defined by Fitch and Budiansky [4], gives the slope of the initial postbuckling path. The parameter ϑ can vary between $+1$ and -1, and it is positive for increasing load, and negative for decreasing load, Fig. 3. Values of ϑ between -1 and $-\frac{1}{2}$ correspond to a backward sloping post-buckling load–deflection curve with decreasing load.

Numerical results

The numerical procedures employed in this study have been described in detail by Akkas and Bauld [5].

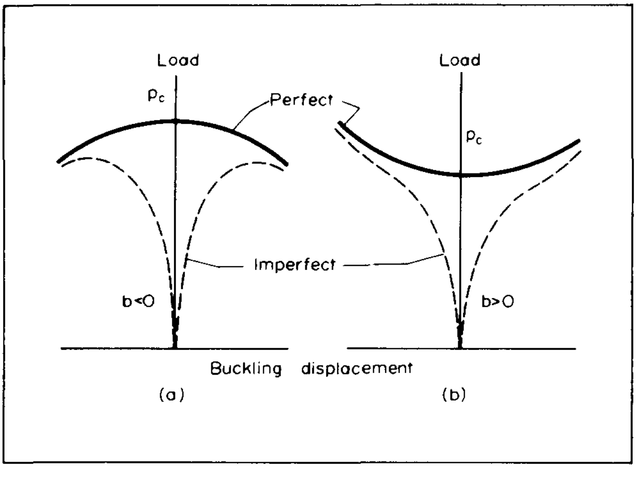

FIG. 2. Interpretation of initial postbuckling coefficient b.

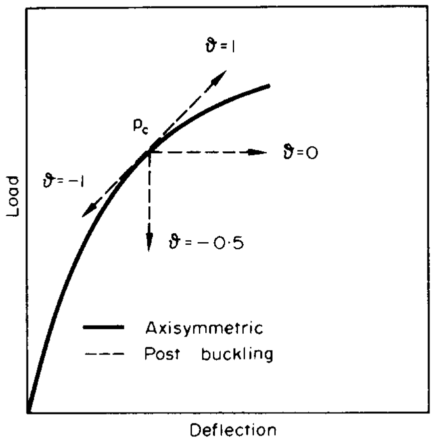

FIG. 3. Interpretation of initial postbuckling relative stiffness parameter ϑ.

Figure 4 shows the effect of the face sheet thickness parameter on the buckling behavior of the clamped shallow spherical sandwich shell under a uniform pressure. For this problem, the geometric shell parameter $\lambda = 15$ and the sandwich shell parameter $\Lambda = 0{\cdot}8$. The core thickness is taken to be equal to the upper face sheet thickness, and the upper and lower face sheets have the same elastic moduli. The nondimensional face sheet thickness parameter is allowed to vary between $\mu = 0{\cdot}5$ and $2{\cdot}0$.

The upper plot in Fig. 4 shows that the bifurcation buckling always precedes the axisymmetric snap-through buckling, at least for the range of μ considered. The critical wave number $n = 4$ for $0{\cdot}50 \leq \mu \leq 1{\cdot}65$ and it is $n = 5$ for $1{\cdot}65 < \mu \leq 2{\cdot}00$. The maximum nondimensional critical load occurs at $\mu = 0{\cdot}8$. Although for $\mu > 0{\cdot}8$ the nondimensional critical load decreases, it should be noted that the physical critical load increases since it is

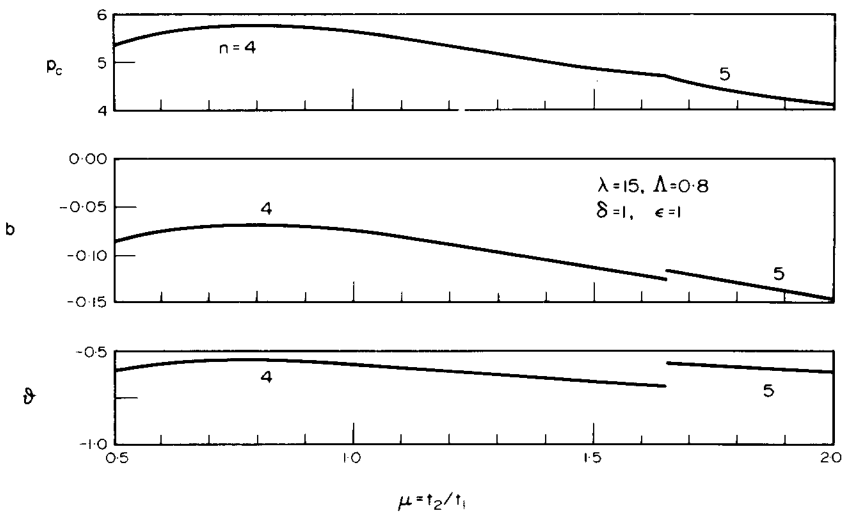

FIG. 4. Buckling and initial postbuckling behavior vs. face sheet thickness parameter.

proportional to $(\mu^4+\mu^3+\mu+1)^{1/2}$. The middle and lower portions of Fig. 4 show that the sandwich cap, for the range of μ considered, is always imperfection sensitive, and the initial postbuckling load-deflection curve has a backward slope; that is, a decrease in deflection accompanies the decrease in load.

The effect of the face sheet material parameter ε on the buckling and initial postbuckling behavior of the sandwich cap with pertinent parameters $\lambda = 15$, $\Lambda = 0{\cdot}8$, $\delta = 1$ and $\mu = 1$ is shown in Fig. 5. For the range of ε considered $(0{\cdot}5 \leq \varepsilon \leq 2{\cdot}0)$, the bifurcation buckling with $n = 4$ always precedes the axisymmetric snap-through buckling and the cap is imperfection-sensitive. The initial postbuckling load-deflection curve has a backward slope.

Figure 6 shows the buckling and initial postbuckling behavior of a clamped shallow spherical sandwich shell with dissimilar face sheets under a uniform pressure distributed over its entire reference surface. The pertinent parameters are $\Lambda = 0{\cdot}8$, $\delta = 1$, $\mu = 2$ and $\varepsilon = 0{\cdot}5$.

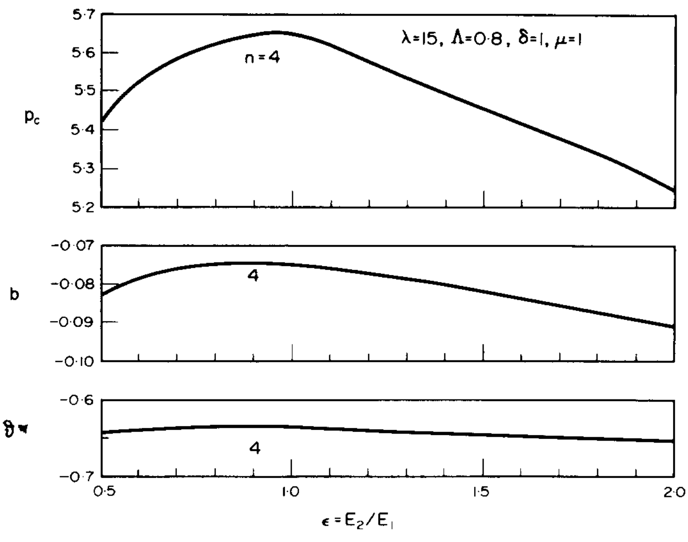

FIG. 5. Buckling and initial postbuckling behavior vs. face sheet material parameter.

The qualitative description of the buckling and initial postbuckling behavior for the sandwich cap with dissimilar face sheets under a uniform pressure is precisely the same as for the homogeneous cap under the same load [3, 4] and also for the sandwich cap with similar face sheets [5]. For $5{\cdot}5 \leq \lambda < 9{\cdot}3$ the buckling behavior is of the axisymmetrical type. For $\lambda \geq 9{\cdot}3$ the buckling behavior is of the asymmetrical type with $b < 0$ which signifies that the asymmetrical buckling process is characterized by a decrease in load carrying capacity for this range of λ. Since the relative stiffness parameter $\vartheta < -0{\cdot}5$ for $\lambda \geq 9{\cdot}3$ it follows that a decrease in deflection accompanies the decrease in load. It was observed, for $\lambda < 5{\cdot}5$, that snap-through buckling did not occur.

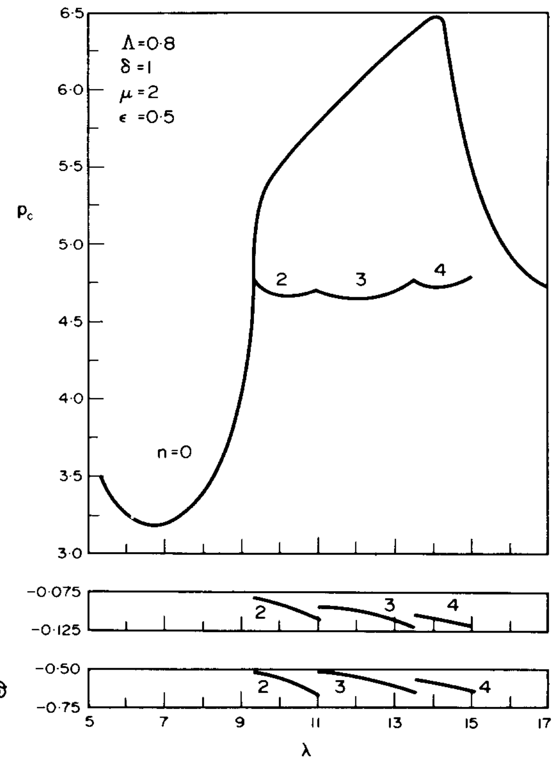

FIG. 6. Buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with dissimilar face sheets.

CLASSICAL BUCKLING ANALYSIS

The classical buckling analysis of a sandwich sphere under a normal pressure has been presented by Yao [8], who concluded that the problem can be treated by using shallow-shell equations because of the fact that the characteristic buckle wavelengths are small compared to the shell radius. Hutchinson, in his analysis of imperfection sensitivity of monocoque spheres [7], employs nonlinear shallow-shell equations. In the present work nonlinear shallow-shell equations will be employed for the buckling analysis of a sandwich sphere under a pressure that is distributed uniformly over the entire reference surface of the sphere.

The equations of nonlinear shallow sandwich shell theory in Cartesian coordinates are derived by Fulton [9] and they are given below:

$$\frac{(E_1t_1^3+E_2t_2^3)}{12(1-\nu^2)}\bar{\nabla}^4 W-\frac{1}{R}\bar{\nabla}^2 F - W_{,xx}F_{,yy} - W_{,yy}F_{,xx}$$

$$+2W_{,xy}F_{,xy}+\frac{Gh^2}{c}(\bar{\alpha}_{,x}+\bar{\beta}_{,y}-\bar{\nabla}^2 W)-q = 0, \tag{12}$$

$$\bar{\nabla}^4 F+(E_1t_1+E_2t_2)\left(\frac{1}{R}\bar{\nabla}^2 W+W_{,xx}W_{,yy}-W^2{}_{,xy}\right) = 0, \tag{13}$$

$$\frac{(1-\nu)}{2}\bar{\nabla}^2\bar{\alpha}+\frac{(1+\nu)}{2}(\bar{\alpha}_{,xx}+\bar{\beta}_{,xy})-\frac{G(1-\nu^2)(E_1t_1+E_2t_2)}{cE_1E_2t_1t_2}(\bar{\alpha}-W_{,x}) = 0, \tag{14}$$

$$\frac{(1-\nu)}{2}\bar{\nabla}^2\bar{\beta}+\frac{(1+\nu)}{2}(\bar{\alpha}_{,xy}+\bar{\beta}_{,yy})-\frac{G(1-\nu^2)(E_1t_1+E_2t_2)}{cE_1E_2t_1t_2}(\bar{\beta}-W_{,y}) = 0, \tag{15}$$

where R denotes the radius of the reference surface of the sandwich shell and

$$h = c+\tfrac{1}{2}(t_1+t_2).$$

The Laplacian operator referred to the Cartesian coordinates is denoted by $\bar{\nabla}^2$. All the other quantities have already been defined.

Prior to buckling the spherical sandwich shell is in a uniform membrane state of stress. To determine the classical buckling pressure q_c, take

$$F = -\tfrac{1}{4}(x^2+y^2)qR+F_1, \tag{16}$$

$$W = qW_0+W_1, \tag{17}$$

$$\bar{\alpha} = \alpha_0+\alpha_1, \tag{18}$$

$$\bar{\beta} = \beta_0+\beta_1, \tag{19}$$

where W_0 is a constant and α_0, β_0 correspond to prebuckling behavior and are zero. Also prior to buckling

$$F_1, W_1, \alpha_1, \beta_1 \text{ are zero.}$$

The linear buckling equations are obtained by substituting F, W, $\bar{\alpha}$ and $\bar{\beta}$ into equations (12–15) and then linearizing with respect to F_1 and W_1. One obtains

$$\frac{(E_1t_1^3+E_2t_2^3)}{12(1-\nu^2)}\bar{\nabla}^4W_1-\frac{1}{R}\bar{\nabla}^2F_1+\frac{1}{2}qR\bar{\nabla}^2W_1+\frac{Gh^2}{c}(\alpha_{1,x}+\beta_{1,y}-\bar{\nabla}^2W_1) = 0, \tag{20}$$

$$\bar{\nabla}^4F_1+\frac{(E_1t_1+E_2t_2)}{R}\bar{\nabla}^2W_1 = 0, \tag{21}$$

and equations (14), (15) on (W_1, α_1, β_1).

Periodic solutions to these homogeneous eigenvalue equations are sought in the form of products of sinusoidal functions such as

$$W_1 = \cos mx \cos ny, \tag{22}$$

$$F_1 = A\cos mx \cos ny, \tag{23}$$

$$\alpha_1 = B\sin mx \cos ny, \tag{24}$$

$$\beta_1 = C\cos mx \sin ny. \tag{25}$$

The eigenvalue associated with this choice, in nondimensional form, is

$$p = \frac{(1+\varepsilon\mu^3)}{4}k+\frac{(1+\varepsilon\mu)}{k}+\frac{3[\delta+\frac{1}{2}(1+\mu)]^2k}{\left[\dfrac{(1+\varepsilon\mu)}{\mu\varepsilon}+\dfrac{24\delta\sqrt{(1+\varepsilon\mu)}}{\Lambda\lambda^2\sqrt{(1+\varepsilon\mu^3)}}k\right]}, \tag{26}$$

where

$$k = (m^2+n^2)Rt_1/\sqrt{[3(1-\nu^2)]}$$
$$p = q/q_0, \qquad q_0 = \frac{2E_1}{\sqrt{[3(1-\nu^2)]}}\left(\frac{t_1}{R}\right)^2. \tag{27}$$

Here, q_0 is the classical buckling pressure of a monocoque full sphere with thickness t_1 and modulus of elasticity E_1. The minimum buckling pressure p_c for the sandwich sphere is found by minimizing p as given by equation (26) with respect to k. This minimization process leads to the following polynomial:

$$k^4 + a_3k^3 + a_2k^2 + a_1k + a_0 = 0, \tag{28}$$

$$a_3 = \{\Lambda\lambda^2[(1+\varepsilon\mu)(1+\varepsilon\mu^3)]^{1/2}\}/(12\delta\mu\varepsilon), \tag{29}$$

$$a_2 = a_3^2/4 - 4(1+\varepsilon\mu)/(1+\varepsilon\mu^3) + \Lambda^2\lambda^4[\delta+\tfrac{1}{2}(1+\mu)]^2/(48\delta^2\mu\varepsilon), \tag{30}$$

$$a_1 = -4(1+\varepsilon\mu)a_3/(1+\varepsilon\mu^3), \tag{31}$$

$$a_0 = a_1a_3/4. \tag{32}$$

The polynomial (28) has only one real positive root, and the classical buckling pressure p_c corresponds to this root.

It is interesting to note that by letting $\mu \to 0$ or $\varepsilon \to 0$, one obtains $p_c = 1$. In other words, if one allows the inner face sheet thickness or elastic modulus to go to zero, then the sandwich sphere reduces to a homogeneous sphere as expected.

By letting $\delta \to 0$, for $\varepsilon = 1$, one obtains $p_c = (1+\mu)^2$; in other words, as $\delta \to 0$ the sandwich sphere reduces to the homogeneous sphere with thickness (t_1+t_2). To see if this theoretical result could be substantiated by a numerical analysis, it was considered to be of interest to study the nature of the buckling and initial postbuckling behavior of the clamped shallow spherical sandwich shell when the core thickness is allowed to go to zero. The pertinent parameters for the shell chosen are $\lambda = 15$, $\Lambda = 0{\cdot}8$, $\mu = 1$ and $\varepsilon = 1$. Via the classical buckling analysis it is conjectured that for this shell, as $\delta \to 0$, the buckling behavior should be of the asymmetric type with $n = 6$ and $p_c = 3{\cdot}10$. Figure 7 shows the effect of the core thickness parameter δ on the buckling and initial postbuckling behavior of the sandwich shell. The conjectured critical load is shown by the little circle on the abscissa of the upper plot in Fig. 7. The core thickness parameter δ is allowed to vary between 10 and 0·1. For $\delta \geq 3{\cdot}40$ the buckling behavior is of the axisymmetric type. For $0{\cdot}35 \leq \delta < 3{\cdot}40$ asymmetric buckling precedes axisymmetrical cap-snapping and the wave numbers associated with the buckling modes increase in succession beginning with $n = 2$ up to $n = 5$ as δ decreases. For $0{\cdot}1 \leq \delta < 0{\cdot}35$ the axisymmetric cap-snapping again precedes the bifurcation buckling. The trend of the buckling curve in Fig. 7 for $0{\cdot}35 \leq \delta \leq 10$ is, in a way, substantiating the conjecture via the classical buckling analysis. The reason for the unexpected apparent buckling behavior for $0{\cdot}1 \leq \delta < 0{\cdot}35$ is unknown; however, it may be suggested that this is due to a numerical instability for these small values of δ.

The middle portion of Fig. 7 gives the initial post-buckling coefficient b as a function of δ. The cap is imperfection sensitive within the range of δ in which the bifurcation buckling precedes the axisymmetric snap-through buckling. It has been suggested in the literature

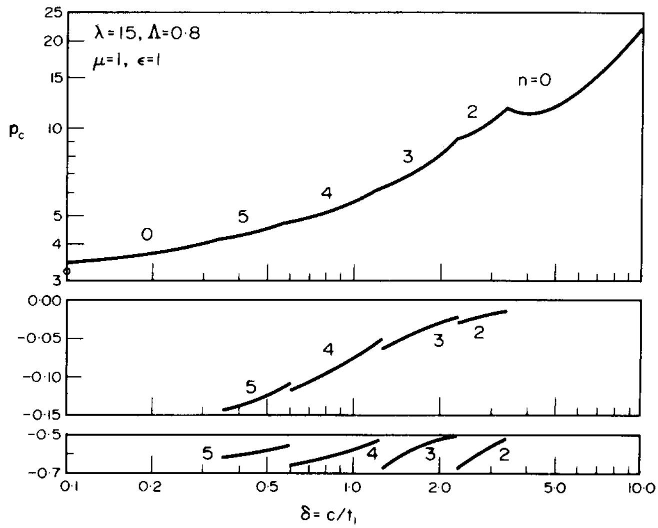

FIG. 7. Buckling and initial postbuckling behavior vs. core thickness parameter.

that as the core thickness of a sandwich shell is increased, its susceptibility to small imperfections is reduced. The results plotted in the middle portion of Fig. 7 are in agreement with this suggestion.

Finally, the lower portion of Fig. 7 shows that, in the range of asymmetric buckling, the initial postbuckling load–deflection curve of the sandwich cap always has a backward slope.

Another interesting observation about equations (26) and (28) is the fact that as the shear modulus of the core material is allowed to go to zero, one obtains, for $\varepsilon = \mu = 1$, $p_c = 2$. That is, as $G \to 0$ the sandwich sphere reduces to two homogeneous spheres one inside the other. Since, in the derivation of the equation, it is assumed that the uniform pressure acts over the reference surface of the sphere, it is obvious that the critical pressure for the sandwich sphere is to be twice that of the monocoque sphere as $G \to 0$ for a given core thickness.

On the other hand, if $G \to \infty$ the critical load becomes, for $\varepsilon = \mu = 1$,

$$p_c = 2\sqrt{(3\delta^2 + 6\delta + 4)}. \tag{33}$$

The effect of the sandwich shell parameter $\Lambda = (G/E_1)(a/H)^2$ on the buckling and initial postbuckling behavior of a clamped shallow spherical sandwich shell subjected to a uniform pressure is shown in Fig. 8. The pertinent parameters are $\lambda = 15$, $\delta = \varepsilon = \mu = 1$. The figure is self-explanatory and the trend of the buckling curve of the upper plot substantiates the results of the classical analysis. Moreover, the middle portion of Fig. 8 shows that one can reduce the degree of imperfection sensitivity of the shell by increasing the shear modulus of the core material.

It is noted from the upper plot of Fig. 8 that for Λ greater than roughly 0·5, increased core shear (i.e. larger values of Λ) does not essentially increase the buckling pressure. For Λ greater than 0·5, at least for the shell under consideration, the assumption of a core with $G = \infty$ is approximately valid. This observation will be utilized in the sequel to relate the geometric parameters of the homogeneous and sandwich shells.

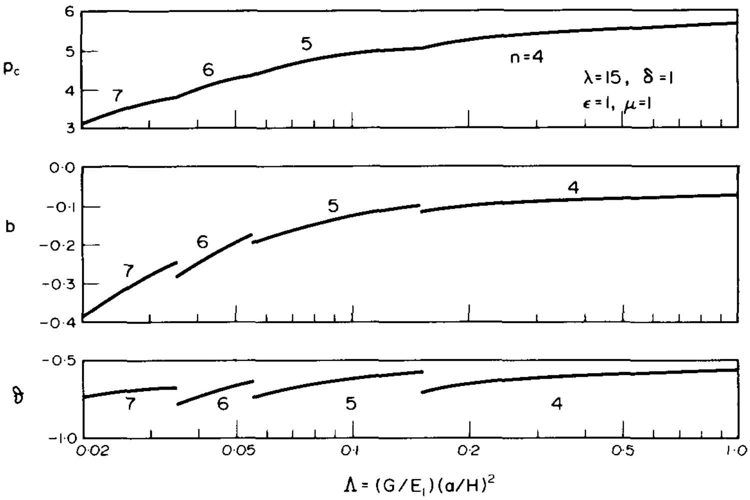

FIG. 8. Buckling and initial postbuckling behavior vs. sandwich shell parameter.

Equations (26) and (28) were solved numerically for two sets of parameters, namely for $\Lambda = 0{\cdot}8$, $\varepsilon = 1$, $\mu = 1$, $\delta = 10$ and for $\Lambda = 0{\cdot}8$, $\varepsilon = 1$, $\mu = 1$, $\delta = 1$. The nondimensional buckling pressure is given as a function of λ in Fig. 9. It should be emphasized that, although here Λ was chosen to be 0·8 for comparison purposes, one need not to specify Λ, since p_c can be plotted as a function of $\Lambda\lambda^2$. In Fig. 9, $p_c = 1$ is the classical buckling pressure of the homogeneous sphere with thickness t_1 and elastic modulus E_1. The classical buckling pressures of the sandwich shells for a given δ increase very rapidly as λ increases, and for the larger values of λ the classical buckling curves are very nearly horizontal lines. In other

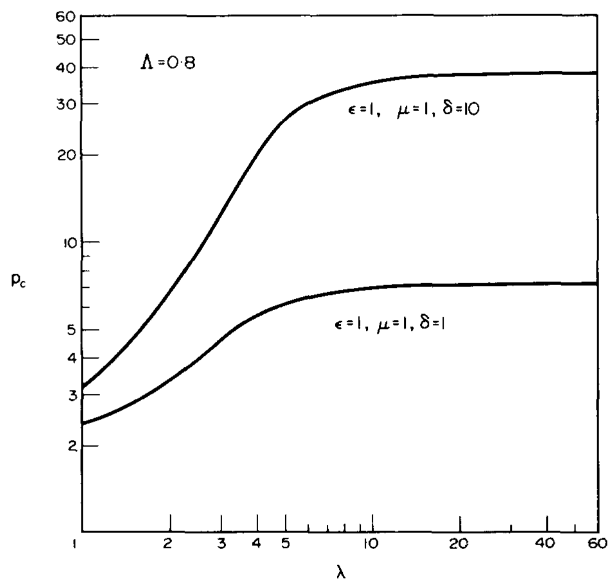

FIG. 9. Classical buckling pressure of sandwich spheres.

words, for these large values of λ, the sandwich shell is essentially behaving like a sandwich shell with a core with infinite shear stiffness.

In Fig. 10 the results of some recent works on the axisymmetric buckling behavior of clamped shallow spherical sandwich shells under a uniform pressure are presented. The asymmetrical buckling curves are not shown; however, the conclusions on the axisymmetrical buckling curves are valid also for the asymmetrical buckling curves.

Curve I in Fig. 10 is the axisymmetrical buckling curve of the spherical homogeneous cap with thickness $2t_1$. It is obtained from the axisymmetrical buckling curve of the classical homogeneous cap (thickness t_1) through the following relations:

$$(\lambda)_I = \sqrt{2}(\lambda)_C, \tag{34}$$

$$(p_c)_I = 4(p_c)_C, \tag{35}$$

where the subscript C corresponds to the classical homogeneous cap.

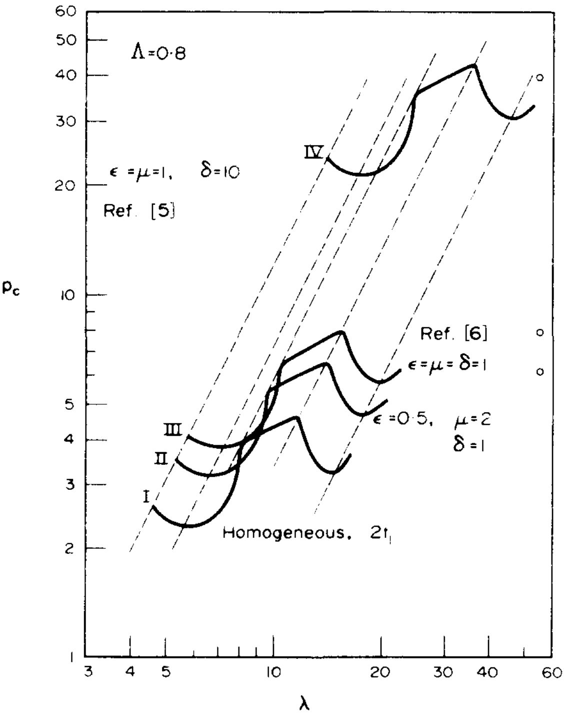

FIG. 10. Comparison of axisymmetric buckling curves of spherical homogeneous and sandwich caps.

The curves I–IV are strikingly similar. In Fig. 10, the dashed lines that connect some characteristic points of the curves are, for all practical purposes, parallel to each other. A study of the numerical data from which the curves in Fig. 10 are constructed indicates that the buckling curves of spherical sandwich shells under a uniform pressure can be obtained almost exactly from that of the corresponding homogeneous shell, via an almost linear transformation, at least for the sandwich shell parameter considered here. Indeed, these data are given in the Appendix for reference purposes for the workers in the field. For all practical purposes, the ratio (magnification factor) between the buckling loads of the sandwich cap and the homogeneous cap for corresponding geometric shell parameters can be determined via the classical buckling analysis of the sandwich spheres. These ratios, in fact, can be obtained from the values shown in circles in Fig. 10.

It was mentioned previously that for Λ greater than roughly 0·5, the assumption of a core with $G = \infty$ is approximately valid. For a sandwich shell with a core with infinite shear stiffness, the following relations hold:

$$\bar{\alpha} = \frac{\partial W}{\partial r}, \qquad \bar{\beta} = \frac{1}{r}\frac{\partial W}{\partial \phi}. \tag{36}$$

For this case, the nondimensional equations (1)–(4) governing the behavior of spherical sandwich shells can be reduced to exactly the form for the homogeneous shell given by Huang [3], but with different nondimensionalization factors. The nondimensional parameters in this case are, for $\varepsilon = 1$ and $\mu = 1$,

$$\bar{\lambda} = 2[3(1-\nu^2)]^{1/4}\left(\frac{H}{t}\right)^{1/2}(3\delta^2+6\delta+4)^{-1/4}, \tag{37}$$

$$\bar{p} = [3(1-\nu^2)]^{1/2}\frac{a^4 q}{16Et^2H^2}[3\delta^2+6\delta+4]^{-1/2}. \tag{38}$$

Using these new parameters in sandwich shell equations, one can, indeed, collapse the curves of Fig. 10 to the single curve for the homogeneous cap.

It should be emphasized that the conclusions of the preceding paragraph are valid only for large values of the sandwich shell parameter Λ, roughly for $\Lambda > 0{\cdot}5$. To see how small core stiffness affects the behavior of spherical sandwich caps, we studied the buckling and initial postbuckling behavior of spherical sandwich caps under a uniform pressure with pertinent parameters $\Lambda = 0{\cdot}2$, $\delta = 3$, $\varepsilon = 1$ and $\mu = 1$. The axisymmetric buckling behavior of this shell has already been presented in Ref. [6]. The results are shown in Fig. 11. The buckling behavior of this sandwich shell is different from that of the homogeneous shell, the difference being more pronounced for smaller values of λ. However, the general conclusions that can be obtained from the curves of Fig. 11 are still similar to those of the homogeneous cap. In other words, the spherical sandwich cap under a uniform pressure with $\Lambda = 0{\cdot}2$ is imperfection-sensitive, the initial postbuckling load-deflection curve has a backward slope and for $\lambda > 12{\cdot}5$ the bifurcation buckling always precedes the axisymmetric snap-through buckling.

CONCLUSIONS

The buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with dissimilar face sheet under a uniform pressure is similar to that of the classical homogeneous cap, at least for the sandwich shell parameter Λ considered here. The susceptibility of a spherical sandwich cap to small imperfections can be reduced by increasing either the core thickness or the shear modulus of the core material. On the basis of the numerical results presented so far, it is concluded that, for large values of Λ, the axisymmetric snap-through buckling curves, hence the asymmetric buckling curves, of spherical sandwich caps can be obtained from those of the classical homogeneous cap. The ratio

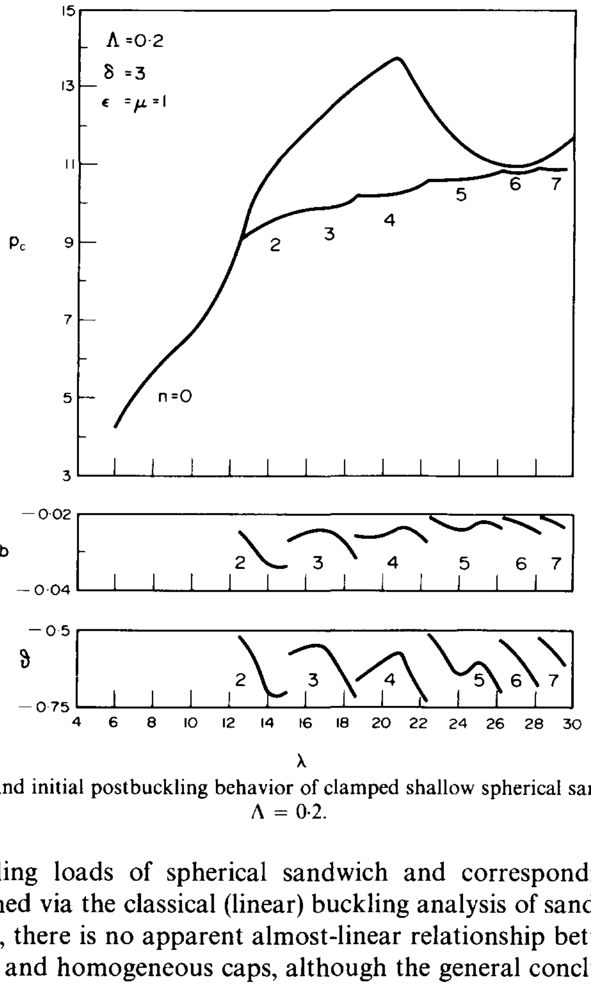

FIG. 11. Buckling and initial postbuckling behavior of clamped shallow spherical sandwich shells with $\Lambda = 0{\cdot}2$.

between the buckling loads of spherical sandwich and corresponding homogeneous shells can be obtained via the classical (linear) buckling analysis of sandwich spheres. For smaller values of Λ, there is no apparent almost-linear relationship between the buckling curves of sandwich and homogeneous caps, although the general conclusions are similar.

Acknowledgements—The financial support for this investigation was provided by the Department of Civil Engineering at Clemson University. The author is indebted to the reviewers for very helpful comments.

REFERENCES

[1] W. T. KOITER, Elastic Stability and Post-Buckling Behavior, *Proc. Symp. Nonlinear Problems*, edited by R. E. LANGER, p. 257. University of Wisconsin Press (1963).

[2] J. R. FITCH, The buckling and post-buckling behavior of spherical caps under concentrated load. *Int. J. Solids Struct.* **4**, 421 (1968).

[3] N. C. HUANG, Unsymmetrical buckling of thin shallow spherical shells. *J. appl. Mech.* **31**, 447 (1964).

[4] J. R. FITCH and B. BUDIANSKY, Buckling and postbuckling behavior of spherical caps under axisymmetric load. *AIAA Jnl.* **8**, 686 (1970).

[5] N. AKKAS and N. R. BAULD, JR., Buckling and initial post-buckling behavior of clamped shallow spherical sandwich shells. *Int. J. Solids Struct.* **7**, 1237 (1971).

[6] N. AKKAS, On the buckling and initial post-buckling behavior of shallow spherical and conical sandwich shells. *J. appl. Mech.* **39**, 163 (1972).

[7] J. W. HUTCHINSON, Imperfection sensitivity of externally pressurized spherical shells. *J. appl. Mech.* **34**, 49 (1967).

[8] J. C. YAO, Buckling of sandwich sphere under normal pressure. *J. aerospace Sci.* **29**, 294 (1962).

[9] R. E. FULTON, Nonlinear equations for a shallow unsymmetrical sandwich shell of double curvature, *Proc. 7th Midwestern Mech. Conf.*, Michigan State Univ., p. 365 (1961).
[10] B. I. HYMAN, Snap-through of shallow clamped spherical caps under uniform pressure. *Int. J. non-linear Mech.* **6**, 55 (1971).

APPENDIX

Axisymmetric buckling loads for clamped shallow spherical homogeneous and sandwich shells under a uniform pressure are given in this Appendix. The notation *NB* implies no snap-through buckling.

TABLE 1

Homogeneous thickness t_1		$\Lambda = 0.8, \delta = 1$ $\mu = 2, \varepsilon = \frac{1}{2}$		$\Lambda = 0.8, \delta = 1$ $\mu = 1, \varepsilon = 1$		$\Lambda = 0.8, \delta = 10$ $\mu = 1, \varepsilon = 1$	
λ	p_c	λ	p_c	λ	p_c	λ	p_c
3·25	*NB*	5	*NB*	5·5	*NB*	13·5	*NB*
3·30†	0·648	5·5	3·42	6	3·96	14	23·54
3·5‡	0·614	6	3·25	7	3·83	15	22·61
4‡	0·578	6·5	3·19	7·5	3·86	16	21·52
4·5	0·580	7	3·21	8	3·94	17	21·30
5‡	0·629	8	3·42	9	4·26	18	21·36
5·5‡	0·762	9	4·05	10	5·14	19	21·66
5·75	0·958	9·5	5·24	10·5	6·36	20	22·18
6‡	0·995	10	5·51	11	6·63	21	22·98
6·5	1·026	11	5·80	11·5	6·82	22	24·20
7‡	1·068	12	6·04	13	7·26	23	26·12
8‡	1·130	13	6·26	15	7·77	24	30·90
8·25	1·136						
8·5	1·046	14	6·48	15·5	7·90	24·5	35·00
9‡	0·935	14·5	5·89	16	7·27	25	35·88
						26	36·90
9·5	0·846	15	5·45	16·5	6·77	30	39·56
10‡	0·825	16	4·94	17	6·42	35	42·34
10·5	0·802	17	4·72	17·5	6·17	35·5	42·60
						36	42·88
11‡	0·835	18	4·66	18	5·99	38	36·94
12‡	0·965	19	4·75	18·5	5·87	40	33·54
						42	31·72
		20	5·09	19	5·79	44	30·80
				19·5	5·75	45	30·60
				20	5·75	46	30·55
				20·5	5·78	48	30·86
				21	5·85	50	31·88

† Reference [10].
‡ Reference [3].

(*Received* 13 *December* 1971; *revised* 24 *April* 1972)

Абстракт—Исследуется выпучивание и начальное поведение после выпучивания пологих заделанных сферических слоистых оболочек, с разными торцевыми слоями, под влиянием одномерного давления. Численные результаты указывают, что выпучивание и начальное поведение после выпучивания заделанной пологой сферической слоистой оболочки, с разными торцевыми слоями, является подобным поведению соответствующей однородной оболочки.

Дается, также, классический анализ выпучивания для сферических слоистых оболочек, под влиянием однородного давления. Результаты указывают на то, что является возможным получить кривые выпучивания сферических слоистых крышок из результатов для однородных крышок, используя увеличение фактора, который получается из классического анализа выпучмвания, для больших значений параметра слоистой оболочки.